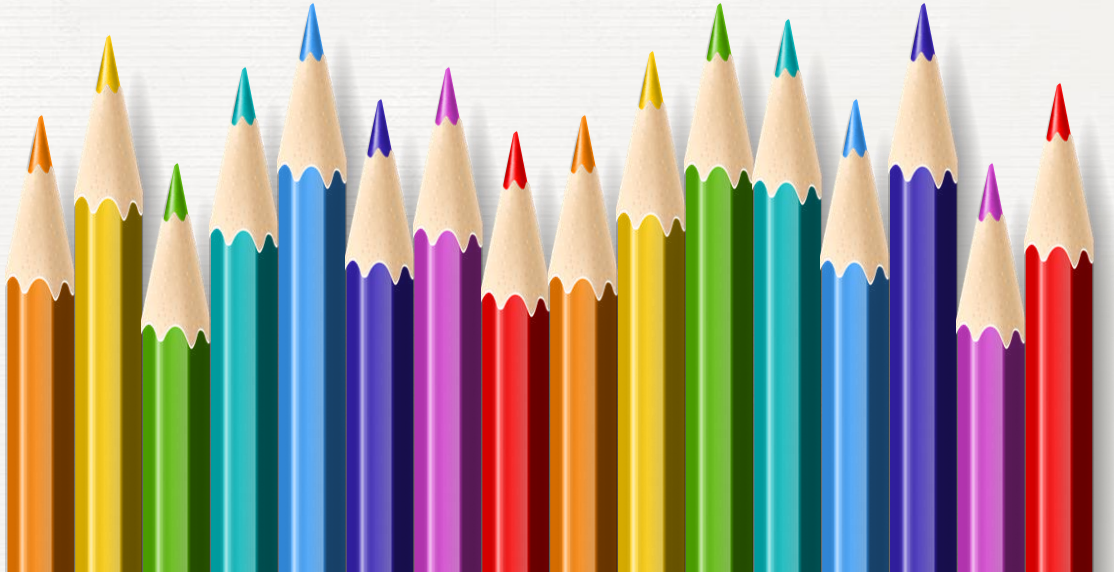
P3 to P5 MATHEMATICS WORKSHOP

FOR PARENTS

Math Heuristics

2 April 2022



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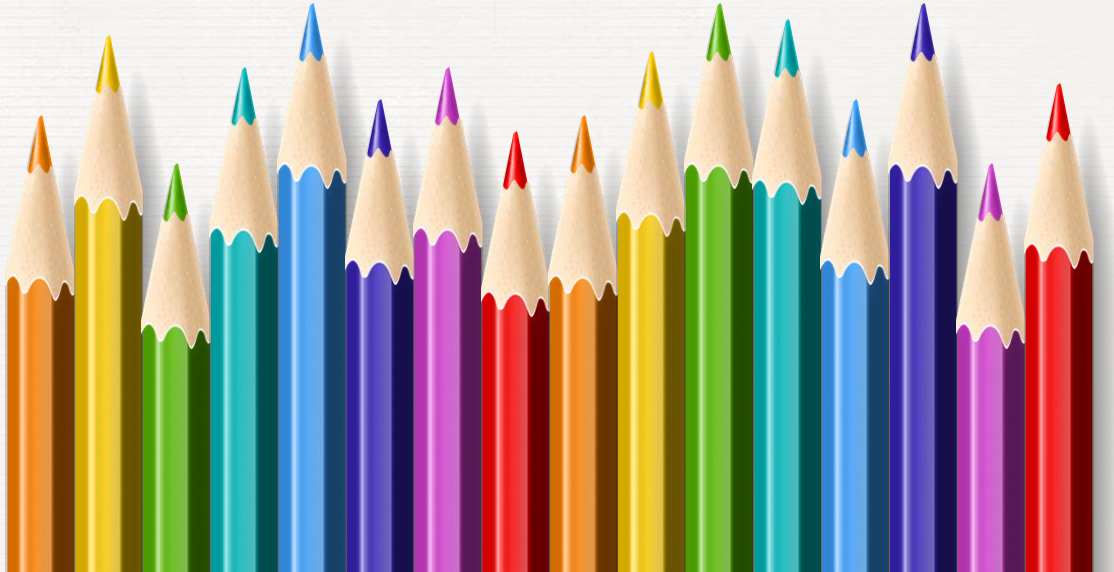


Programme

0945 – 0955 → **Objectives and Overview**

0955 – 1045 → **Problem Solving Strategies**

- Guess and Check
- Model Drawing

(Screen Break)

- Working Backwards

1045 – 1100 → **Feedback & Q n A**

Objectives for this workshop

1) Empower parents with the necessary skills to guide their children with the different strategies.

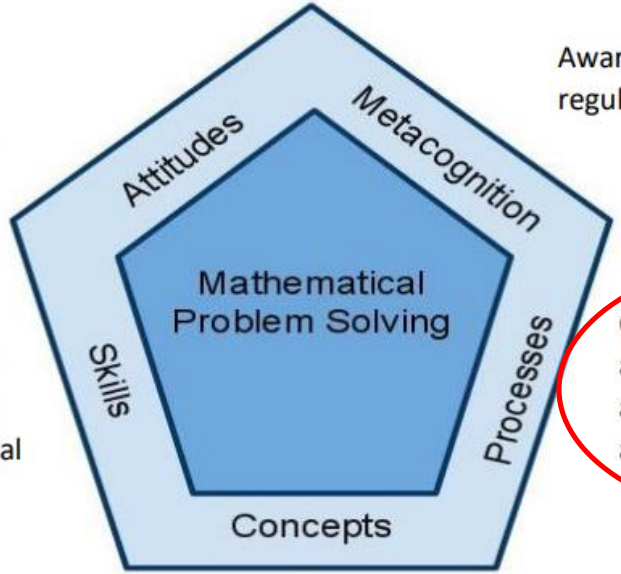
2) Enhance parents' role as supportive partners.

Mathematics Framework

Mathematics Curriculum Framework

Belief, appreciation, confidence, motivation, interest and perseverance

Awareness, monitoring and regulation of thought processes



Proficiency in carrying out operations and algorithms, visualising space, handling data and using mathematical tools

Competencies in abstracting and reasoning, representing and communicating, applying and modelling

Understanding of the properties and relationships, operations and algorithms

What is Mathematical Problem Solving?

It is the **process** in which a person, when given a mathematical problem, applies mathematical **concepts, skills and processes** to solve the problem.

It involves **designing** and **carrying out a set of steps** to reach the **final solution**.

Steps in Mathematical Problem-solving

POLYA's Problem Solving Approach and DEAL

1. READ and UNDERSTAND
(Define the problem)

- What is the problem?
- What are we trying to find out?
- Can we restate the problem?

2. PLAN
(Explore the information)

- What do we know ?
- What do we need to do to solve the problem?
- Do we need more information?
- Is there a hidden question?

3. DO
(Act on it)

- Carry out the plan.

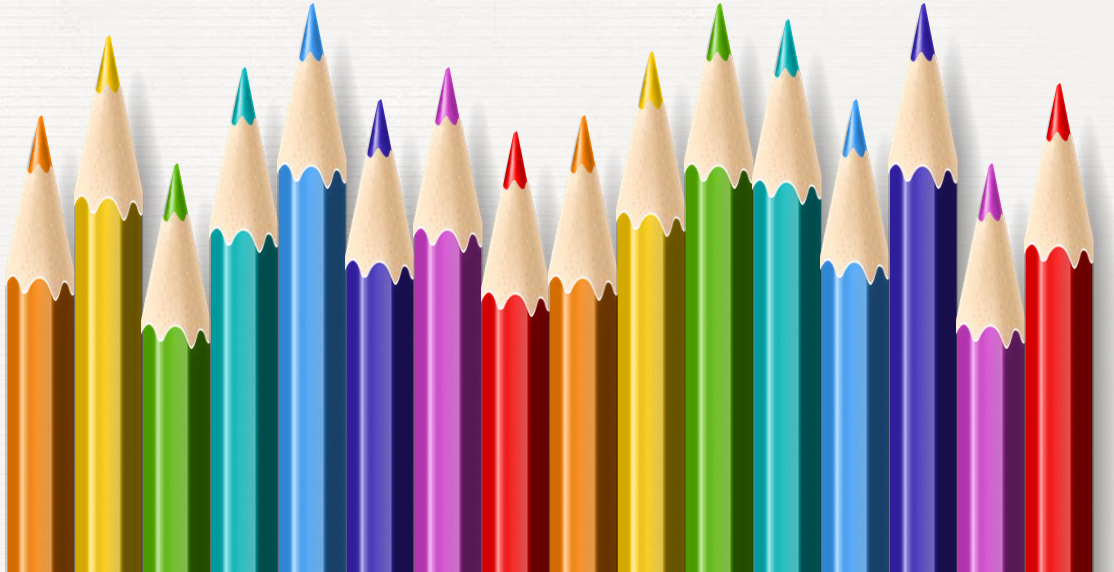
4. CHECK
(Look back and check)

- Compare solution with the original question.
- Does it make sense?
- Should we revise our plan to meet all conditions?

Strategies covered today:

- ★ Guess and Check
- ★ Working backwards
- ★ Model Drawing

Guess and Check



Guess and Check

- It is a way to make **logical guesses** in a **systematic** manner.
- If the calculated total is **less than the targeted total**, **increase** the number which has a **greater value**. Following that, observe the gap again.
- To begin using the **Guess and Check** Method, we usually start with an **equal amount on both sides**.

*Guess and Check (P3)

Tip: Start guessing with half of the total number and increase/ decrease accordingly (making a calculated guess).

There are 12 chickens and goats in a farm.
There are a total of 42 legs altogether.
How many chickens are there in the farm?

Chickens	Chicken Legs	Goats	Goat Legs	Total number of Animals	Total number of Legs	Check
6	6 x 2 = 12	6	6 x 4 = 24	12	12 + 24 = 36	✗
4	4 x 2 = 8	8	8 x 4 = 32	12	8 + 32 = 40	✗
3	3 x 2 = 6	9	9 x 4 = 36	12	6 + 36 = 42	✓

Tip: Organise the guesses in a table for easy checking.

There are 3 chickens in the farm.

**Guess and Check (P3)

Tip: The guesses must be based on the given conditions

There are 12 bicycles and tricycles in a park.
There are a total of 32 wheels altogether.
How many bicycles are there in the park?

Bicycles	Bicycle Wheels	Tricycles	Tricycle Wheels	Total number of Bicycles and Tricycles	Total number of Wheels	Check
6	6 x 2 = 12	6	6 x 3 = 18	12	12 + 18 = 30	✗
5	5 x 2 = 10	7	7 x 3 = 21	12	10 + 21 = 31	✗
4	4 x 2 = 8	8	8 x 3 = 24	12	8 + 24 = 32	✓

Important: A minimum of 3 guesses are needed to ascertain that the conditions are fulfilled consistently.

There are 4 bicycles in the park.

Guess and Check (P4)

Tip: Master the multiplication tables to relieve cognitive load.

Joan had **20 coins**. There were **five-cent and fifty-cent coins**.
She had a total of **$7.30**. How many **five-cent coins** did Joan have?

5-cent coins		50-cent coins		Total		Check
No.	$	No.	$	No.	$	
10	10 x $0.05 = $0.50	10	10 x $0.50 = $5	20	$0.50 + $5 = $.5.50	✗
8	8 x $0.05 = $0.40	12	12 x $0.50 = $6	20	$0.40 + $6 = $6.40	✗
6	6 x $0.05 = $0.30	14	14 x $0.50 = $7	20	$0.30 + $7 = $7.30	✓

Check: The guesses must be based on the given conditions

Joan had 6 five-cent coins.

Guess and Check (P4)

Questions may seem more complex but the same strategy is used to solve.

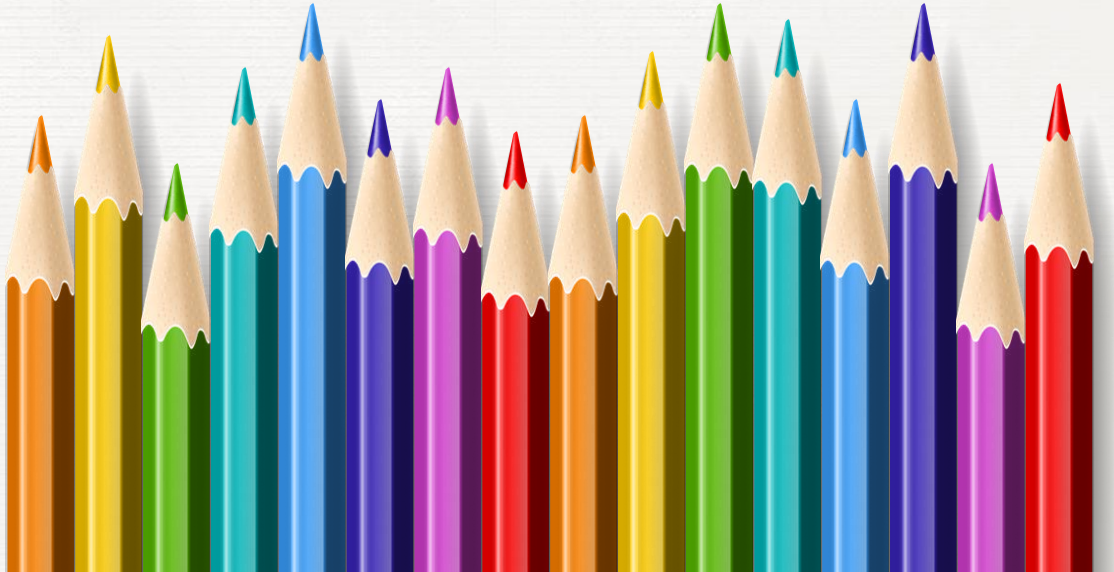
During a competition, **8 points were awarded for every gold medal** won and **6 points were awarded for every silver medal won**.

Given that a total of **512 points** were awarded for **70 medals**, find the number of silver medals won during the competition.

Gold		Silver		Total		Check
No.	Points	No.	Points	No.	Points	
35	35 x 8 = 280	35	35 x 6 = 210	70	280 + 210 = 490	✗
40	40 x 8 = 320	30	30 x 6 = 180	70	320 + 180 = 500	✗
46	46 x 8 = 368	24	24 x 6 = 144	70	368 + 144 = 512	✓

There were 24 silver medals won during the competition.

Model Drawing



Model Drawing

- It is a way to help students visualise and understand the concepts working from a concrete concept to an abstract .

An example:

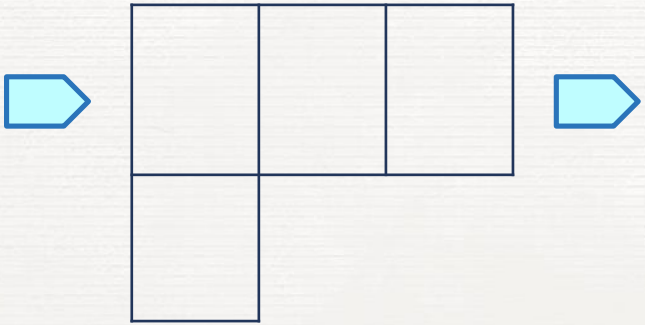
Concrete

3 cupcakes

1 cupcake

Pictorial

3 cupcakes



1 cupcake

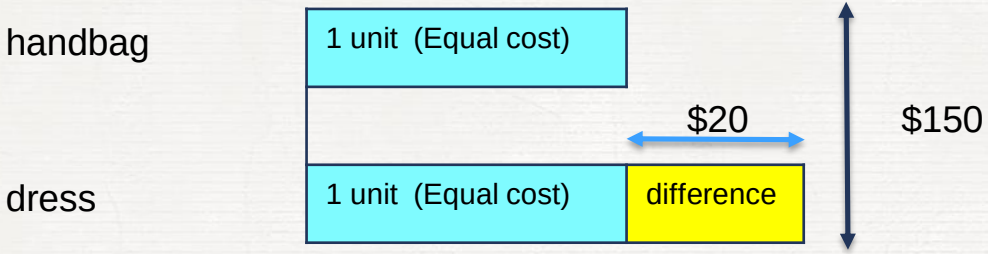
Abstract

3 cupcakes

1 cupcake

Model Drawing (P3)

Mrs Tan spent $150 on a handbag and a dress. The handbag cost $20 less than the dress. How much did Mrs Tan spend on the handbag?



This is an example of a P3 question involving model drawing and the concept of more than/ less than.

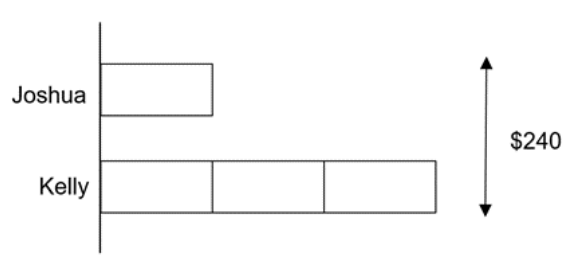
2 units = 150 – 20
= 130

1 unit = 130 ÷ 2
= 65

The handbag cost $65.

*Model Drawing (P3)

Both Kelly and Joshua have a total of 240 cards. Kelly had **3 times as many cards** as Joshua. How many more cards did Kelly have than Joshua?



Method 1:

4 units = 240

1 unit = 240 ÷ 4

= 60 **(Joshua)**

3 units = 60 x 3

= 180 **(Kelly)**

Difference = 180 – 60

= 120

Kelly has 120 more cards

Method 2:

From the model, we can tell that Kelly has 2 more units than Joshua.

Therefore,

Difference = 180 – 60

= 120

Kelly has 120 more cards

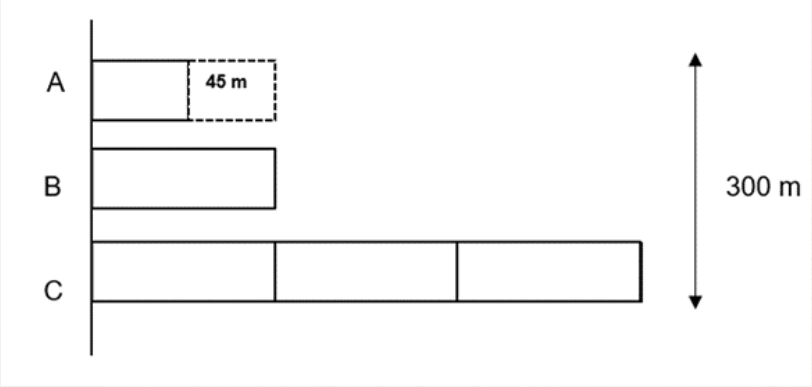
This is an example of a P3 question involving model drawing and unitary method.

Common mistake here is students simply divide by 3 instead of 4 because without the model drawing they are not able to visualise.

So it is best that they draw out their model for better clarity.

**Model Drawing (P4)

A 300 m ribbon is cut into three pieces A, B and C.
A is 45 m shorter than B. C is 3 times as long as B. How long is B?



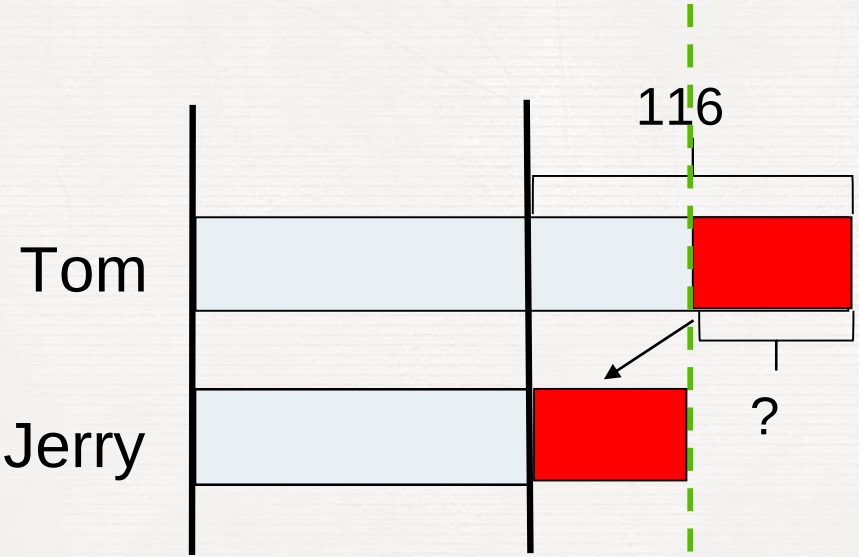
5 units = 300 + 45
= 345
1 unit = 345 ÷ 5
= 69

Ribbon B is 69 m.

**300 m excludes the 45 cm.
By including 45 cm, the students will be able to find the total value of 5 units.
Without the model, it is again not easy for students to visualise.
With the model, the correlation between the number of units and total becomes clearer.**

Model Drawing (P4)

Tom and Jerry had $240 and $124 respectively. How much must Tom give to Jerry so that they have the same amount of money ?



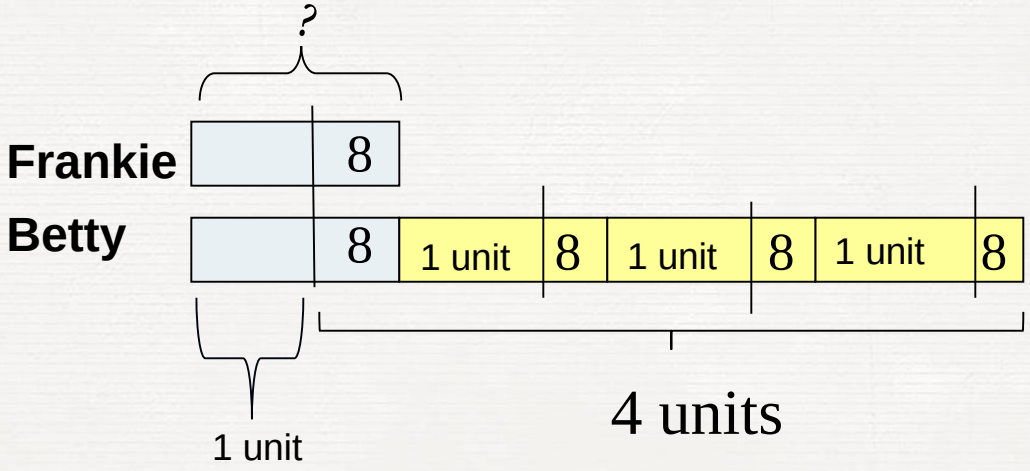
240 – 124 = 116

116 ÷ 2 = 58

Tom must give $58 to Jerry.

Model Drawing (P5)

Frankie and Betty had an equal amount of money.
After Betty received thrice the amount of money that she had at first, and Frankie spent $8, Betty had 5 times as much money as Frankie.
How much money did Frankie have at first?



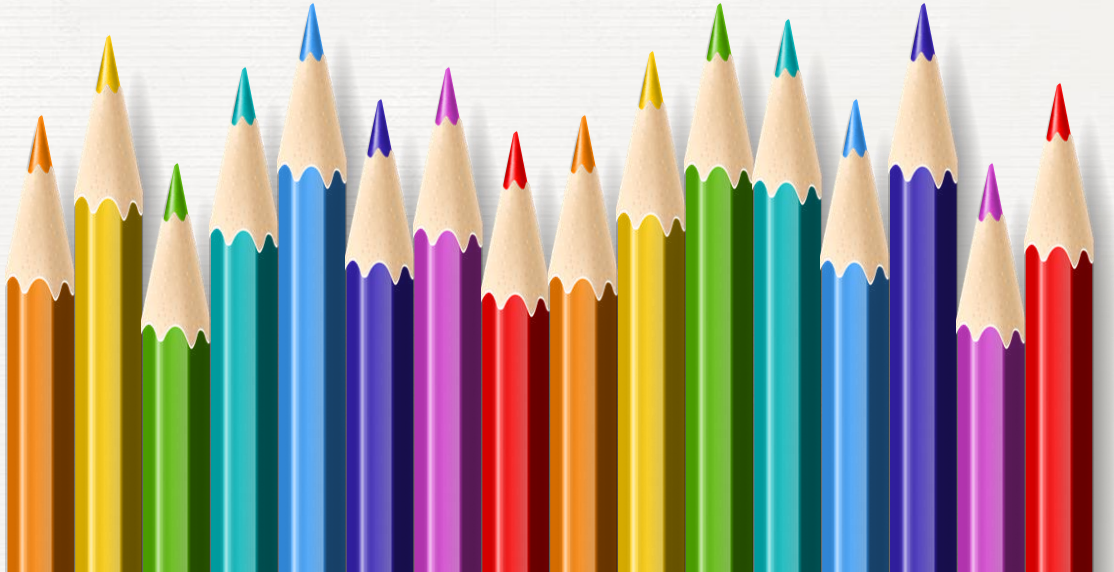
1 unit = 4 x 8

= 32

32 + 8 = 40

Frankie had $40 at first.

Working Backwards



Working Backwards

- start with the answer at the end and work backwards to find the missing information.

- decide which operation to use when working backwards in order to solve the problem.

Working Backwards is a strategy students can use to check their answers for accuracy and reasonableness. This is also a part of the DEAL process, where L stands for Look back and check.

How do we identify questions that need working backwards?

1) In the question, the number of people or objects at first is unknown.

2) A series of events takes place.

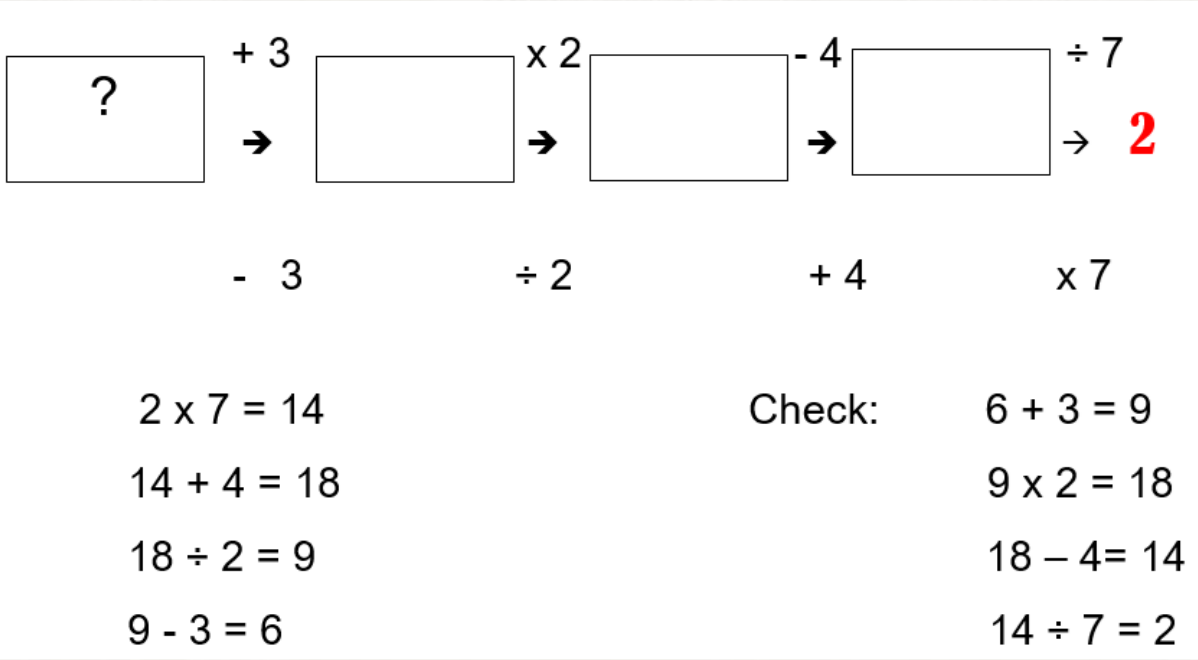
3) The final number is given at the end of the question situation.

4) The question requires to find the beginning number.

* Working Backwards (P3)

I think of a number and add 3 to it, multiply the result by 2, subtract 4 and divide by 7.

The number I end up with is 2. What is the number I first thought of?



To work backwards, you reverse the operations, from the end to the start.

** Working Backwards (P4)

There were some passengers on a bus. At bus stop A, 5 passengers alighted and 8 passengers boarded. At bus stop B, 6 passengers alighted. Then there were 70 passengers. How many passengers were there on the bus at first?

Clue : Use the end number and work backwards to arrive at the number of passengers at first. (Start with 70 passengers)

** Working Backwards

There were some passengers on a bus. At bus stop A, 5 passengers alighted and 8 passengers boarded. At bus stop B, 6 passengers alighted. Then there were 70 passengers. How many passengers were there on the bus at first?

Use the end number and work backwards to arrive at the number of passengers at first. (Start with 70 passengers)

70 + 6 = 76

76 - 8 = 68

68 + 5 = 73

Check: **73 - 5 = 68**

68 + 8 = 76

76 - 6 = 70

Working Backwards (P5- involving Fractions)

Bags A, B and C contain 540 beads altogether. $\frac{2}{5}$ of the beads in Bag A were put into Bag B. Then, $\frac{1}{4}$ of the beads in Bag B were put into Bag C. Finally, $\frac{1}{3}$ of the beads in Bag C were put into Bag A. There was an equal number of beads in each bag in the end. How many beads did each bag contain at first?

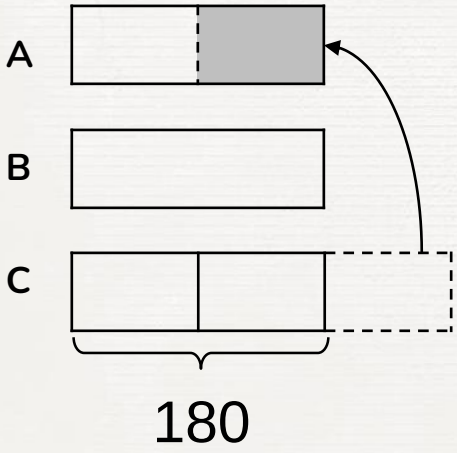
Working Backwards(P5)

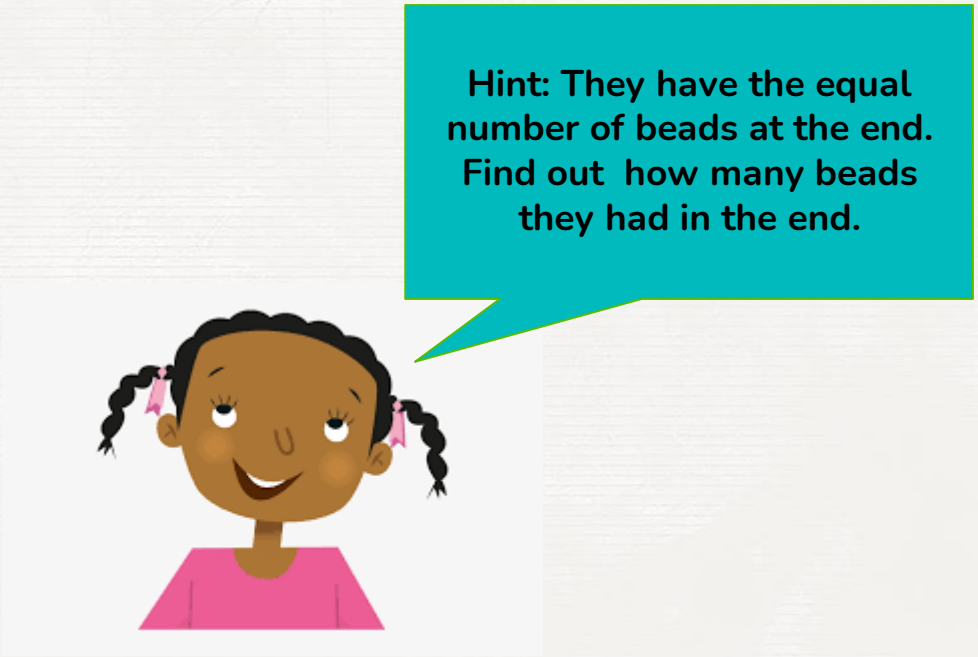
Bags A, B and C contain 540 beads altogether. $\frac{2}{5}$ of the beads in Bag A were put into Bag B. Then, $\frac{1}{4}$ of the beads in Bag B were put into Bag C. Finally, $\frac{1}{3}$ of the beads in Bag C were put into Bag A. There was an equal number of beads in each bag in the end. How many beads did each bag contain at first?

$540 \div 3 = 180$

Each bag contained 180 beads in the end.

Last step ($\frac{1}{3}$ of C to A):





Working Backwards (P5)

Stage	Working	Number of beads in A	Number of beads in B	Number of beads in C
In the end (after C to A)		180	180	180
Before C to A	2 units = 180 1 unit = 90	180 – 90 = 90	180	180 + 90 = 270
Before B to C	3 units = 180 1 unit = 60	90	180 + 60 = 240	270 – 60 = 210
Before A to B	3 units = 90 1 unit = 30 2 units = 60	90 + 60 = 150	240 – 60 = 180	210

There were **150, 180 and 210** beads in Bags A, B and C respectively at first.

Parents' Help with Child At Home

Math in real life

- Talk to your children about mathematics and help them to see the usefulness of math in our daily lives.
- Addition and subtraction of money at the supermarket, involve them in estimating the total cost
- Multiply in sets, eg. 5 apples cost $4, 20 apples cost $16
- Look at patterns around housing estates and buildings.
- Commit Math facts to memory - practise multiplication and division, eg. multiply the digits found on car plates
- Expose your children to different occupations which use maths in their daily job scope- math skills and concepts are meaningful purposeful and essential

“The essence of Mathematics is not to make simple things complicated, but to make complicated things simple” (Mathematician Stan Gudder)